Second-Harmonic Generation by Electromagnetic Waves at the Surface of a Semi-Infinite Metal

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Abstract— Reference [1], which appears in the Proceedings of the RADAR'09 Conference at Bordeaux in October 2009, describes nonlinear currents generated by an electromagnetic wave incident on a metal surface. This paper supplements the results of [1] with a full calculation of the RCS per unit area for second harmonic generation. These results verify that the level of such emission from a metal surface is anomalously low.

Keywords –second harmonic; scattering; metal physics; radar nonlinear response; radar cross section.

I. INTRODUCTION

Current and future proposals to use radar in situations that extend beyond its traditional role as a method of remote sensing, e.g., airport security (weapons detection), mapping (SAR), and mine detection, are often constrained by the need to overcome certain limitations of its capabilities. When this situation exists, various marginal technologies need to be re-examined periodically with a goal of addressing such needs. One such technology is harmonic and multitone radar, in which a signal illuminating a target at one frequency creates a response at another frequency. The advantage of this type of radar lies in its frequency diversity – out-of-band returning signals may escape jamming, clutter and multipath effects can be mitigated, etc. In addition, the receiver may be able to identify new properties of the target, i.e., novel signatures. This paper will focus on the last possibility.

In many respects, understanding the nonlinear response of a target, which was the focus of our paper Ref. [1], is primarily a problem of material science rather than radar technology. In Ref. [1] we investigated the ability of a signal to induce nonlinear currents at a metal surface, without investigating the ability of these currents to radiate energy back into the vacuum. In this paper we complete this task by computing the fields radiated by the second-harmonic currents, and present results for the electromagnetic fields radiated into vacuum by these currents.

II. BARE METAL SURFACES

As discussed in [1], electromagnetic fields are difficult to create within a metal due to screening by its mobile electrons, which confines the incident fields to a region near the metal surface whose thickness is the metallic skin depth.[2] Within the skin depth the fields attenuate rather than propagate, causing the metal to reflect electromagnetic energy efficiently. At radar frequencies this high reflectivity obeys the Hagen-Rubens law [3]:

$$\Re = -1 + (1-i)\sqrt{\frac{2\omega}{\omega_p{}^2\tau}} \quad ,$$

where $\omega_p \approx 10^7$ GHz is the metal plasma frequency and τ is its conductivity (Maxwell) relaxation time. In copper we find that $\omega_p \approx 10^7$ GHz and $\omega_p\tau \approx 320$, so that the reflectivity at 10 GHz is effectively –1, i.e., total reflection apart from a correction of order 10^{-4}, which is in fact the transmittivity into the metal. In addition, since the skin depth of copper at 10 GHz is only 60 microns, it is clear that nonlinear effects are strongly localized at the surface.

In [1] we discussed a well-established model for the metal electron "gas" based on plasma physics [4], where the motion of mobile electrons obeys the equations of motion for a charged fluid (i.e., a plasma). Because the radar signal is never very strong at a target, it is legitimate to solve these equations in a perturbation series. The various terms in the perturbation series generate new frequencies in the induced target current, which radiate energy back to the receiver.

III. ILLUMINATION IN THE LINEAR REGIME

Let us begin with the case of a weak EM field incident on a metal surface. . It is well known [5] that in order to specify the problem of off-normal incidence of an EM wave on a surface, we must first define a plane of incidence, which contains both the wave vector of the incident wave and the surface normal. Then an arbitrary wave polarization will be a linear combination of two components:

(1) Magnetic field in the plane of incidence and electric field perpendicular to it (*s* – polarization)

(2) Electric field in the plane of incidence and magnetic field perpendicular to it (*p* – polarization)

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14. ABSTRACT
Reference [1], which appears in the Proceedings of the RADAR?09 Conference at Bordeaux in October 2009, describes nonlinear currents generated by an electromagnetic wave incident on a metal surface. This paper supplements the results of [1] with a full calculation of the RCS per unit area for second harmonic generation. These results verify that the level of such emission from a metal surface is anomalously low.

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The geometry of this problem is shown in Fig. 1 for a *p* – polarized wave. This polarization tends to have the strongest interaction with the metal since the electric field generally has a component perpendicular to the surface. For this case the linear EM fields outside the metal ($z > 0$) are

$$\vec{E} = E_0\left[e^{-ikz\cos\theta}\hat{\varepsilon}_i + \Re e^{ikz\cos\theta}\hat{\varepsilon}_r\right]e^{ikx\sin\theta}e^{-i\omega t},$$
$$\vec{B} = \left[-\frac{1}{c}E_0\left(e^{-ikz\cos\theta} - \Re e^{ikz\cos\theta}\right)\hat{y}\right]e^{ikx\sin\theta}e^{-i\omega t}$$

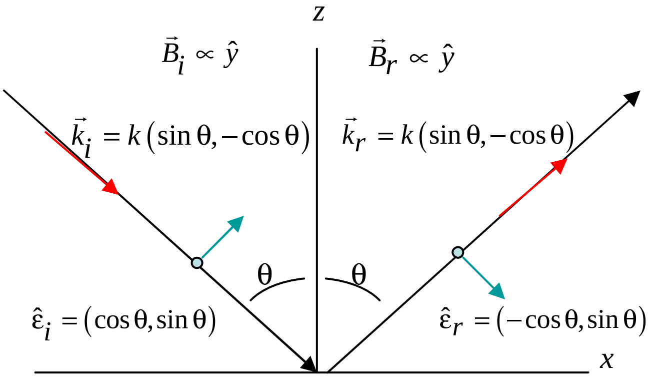

Figure 1: *p* – polarized incident wave geometry; the unit vectors $\hat{\varepsilon}_r$ are polarization (electric-field) vectors. Vectors are written in rectangular component form, i.e., (Ax , Az)

where E_0 is the incident electric field, $\Re$ is the reflection coefficient of the surface, $k = \omega / c$, and θ is the angle of incidence measured form the surface normal. Within the metal, the wave properties are modified by the presence of mobile charges. As discussed in [1], combining the plasma equations with the Maxwell equations leads to both electrical and mechanical degrees of freedom in the metal, which are coupled in such a way that the electrical oscillations in the metal fall into two categories [6]:

(1) Electromagnetic oscillations with nonzero magnetic field and zero density oscillations (referred to as "transverse plasmons"), and
(2) Electrostatic oscillations with zero magnetic field and nonzero density oscillations (referred to as "longitudinal plasmons")

Dissipation causes such oscillations to be attenuating in time when free and evanescent in space when driven by a field from outside the metal. The additional mechanical degrees of freedom (electron density and velocity) imply that the usual Maxwell boundary conditions, i.e., continuity of tangential $\vec{E}$ and $\vec{B}$, are insufficient to define the problem in the presence of the surface, and require an additional boundary condition, which in the metal-physics literature is taken to be the condition that there be no electron velocity perpendicular to the surface.[7] This condition is independent of the usual Maxwell boundary conditions, and leads to dispersive surface waves (called "surface plasmons") that have been studied extensively.

Within the metal ($z < 0$) the fields take the form

$$\vec{E} = \left[Ae^{-Pz}\left(\hat{x} + \frac{ik\sin\theta}{P}\hat{z}\right) + Be^{-Qz}\left(\frac{ik\sin\theta}{Q}\hat{x} - \hat{z}\right)\right]e^{ikx\sin\theta}e^{-i\omega t},$$
$$\vec{B} = \left[\frac{k}{icP}\left\{1 - \frac{\omega_p^2}{\omega\left(\omega + \frac{i}{\tau}\right)}\right\}Ae^{-Pz}\hat{y}\right]e^{ikx\sin\theta}e^{-i\omega t}$$

where *A* and *B* are determined by the continuity conditions plus the extra boundary condition. The angle-dependent attenuation exponents are given by

$$P^2 = k^2\sin^2\theta + \frac{1}{c^2}\left\{\frac{\omega}{\omega + \frac{i}{\tau}}\omega_p^2 - \omega^2\right\}, \qquad (1)$$
$$Q^2 = k^2\sin^2\theta + \frac{3}{\upsilon_F^2}\left\{\omega_p^2 - \omega\left(\omega + \frac{i}{\tau}\right)\right\}$$

where

$$\omega_p = \frac{n_0 e^2}{\varepsilon_0 m} \qquad \upsilon_F = \frac{\hbar}{m}\left(3\pi^2 n_0\right)^{1/3}$$

are the "plasma frequency" and Fermi velocity of the metal.[8] Some manipulation of the fluid equations shows that the current is given by

$$\vec{j}_1 = \frac{\varepsilon_0}{-i\omega + \frac{1}{\tau}}\left[\omega_p^2\vec{E} - \frac{3}{\upsilon_F^2}\nabla\left(\nabla\cdot\vec{E}\right)\right].$$

Using the expression for $\vec{E}$ inside the metal, we find that $j_{1z} = i\omega\varepsilon Be^{-Pz}$, so that the boundary condition $j_{1z}(z=0) = 0 \Rightarrow B = 0$. The constant *A* can then be written $\Im E_0$, thereby defining a transmission coefficient $\Im$ along with $\Re$. Applying the electromagnetic boundary conditions then gives us

$$\Im(\theta) = \frac{2\cos\theta}{\left\{1 - \frac{\omega_p^2}{\omega\left(\omega + \frac{i}{\tau}\right)}\frac{(k\sin\theta)^2}{P(\theta)Q(\theta)}\right\} + \left\{1 - \frac{\omega_p^2}{\omega\left(\omega + \frac{i}{\tau}\right)}\right\}\frac{i\omega}{cP(\theta)}\cos\theta}$$

and

$$\Re(\theta)=\frac{\left\{1-\frac{\omega_p^2}{\omega\left(\omega+\frac{i}{\tau}\right)}\frac{(k\sin\theta)^2}{P(\theta)Q(\theta)}\right\}-\left\{1-\frac{\omega_p^2}{\omega\left(\omega+\frac{i}{\tau}\right)}\right\}\frac{i\omega}{cP(\theta)}\cos\theta}{\left\{1-\frac{\omega_p^2}{\omega\left(\omega+\frac{i}{\tau}\right)}\frac{(k\sin\theta)^2}{P(\theta)Q(\theta)}\right\}+\left\{1-\frac{\omega_p^2}{\omega\left(\omega+\frac{i}{\tau}\right)}\right\}\frac{i\omega}{cP(\theta)}\cos\theta}.$$

These expressions solve the linear problem completely, which initializes the perturbation calculation.

IV. NONLINEAR CURRENTS

The second-order equation for the electric field is an inhomogeneous version of the linear equation of the form

$$\begin{aligned}&\left(\frac{\partial}{\partial t}+\frac{1}{\tau}\right)\left(\nabla^2\vec{E}_2-\nabla\left(\nabla\cdot\vec{E}_2\right)-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E}_2\right)\\&-\frac{1}{c^2}\frac{\partial}{\partial t}\left(-\frac{\upsilon_F^2}{3}\nabla\left(\nabla\cdot\vec{E}_2\right)+\omega_p^2\vec{E}_2\right)\\&=\mu_0\frac{\partial}{\partial t}\left\{\begin{array}{l}\varepsilon\omega_p^2\vec{\upsilon}_1\times\vec{B}_{s1}+\left(\frac{\partial}{\partial t}+\frac{1}{\tau}\right)\left(-en_1\vec{\upsilon}_1\right)\\+en_0\vec{\upsilon}_1\cdot\nabla\vec{\upsilon}_1-\frac{\upsilon_F^2}{9n_0}en_1\nabla n_1\end{array}\right\}=\vec{S}\end{aligned},$$

where $\vec{S}=\vec{S}_1+\vec{S}_2+\vec{S}_3+\vec{S}_4$ are source terms arising from the four nonlinear processes discussed in [1]. For an incident single-frequency p – polarized plane wave we can introduce the usual complex notation in which $\vec{E}=\mathrm{Re}\,\vec{\mathrm{E}}$ and $\vec{S}=\mathrm{Re}\,\vec{\Sigma}$, in which case these inhomogeneous currents become:

$$\begin{aligned}\vec{\Sigma}_1=&-\frac{i\omega e}{m}\frac{3\omega_p^2}{\upsilon_F^2}\frac{1}{c^2}\frac{2\omega+\frac{i}{\tau}}{\omega+\frac{i}{\tau}}\left[1-\frac{\omega_p^2}{\omega\left(\omega+\frac{i}{\tau}\right)}\right]\frac{ik\sin\theta}{PQ}\\&\cdot E_0^2\Im^2e^{-Qz}\left\{\begin{array}{l}\left(\hat{x}\frac{ik\sin\theta}{Q}-\hat{z}\right)\frac{ik\sin\theta}{P}e^{-Qz}\\+\left(\hat{x}+\frac{ik\sin\theta}{P}\hat{z}\right)e^{-Pz}\end{array}\right\}e^{2ikx\sin\theta}\end{aligned}$$

$$\begin{aligned}\vec{\Sigma}_2=&\frac{i\omega e}{m}\frac{\omega_p^2}{c^2}\left\{\frac{1}{\omega+\frac{i}{\tau}}\right\}^2 ik\sin\theta\\&\cdot E_0^2\Im^2e^{-Qz}\left\{\begin{array}{l}\hat{x}\left(\frac{(ik\sin\theta)^2}{PQ}e^{-Qz}+e^{-Pz}\right)\left(\frac{(ik\sin\theta)^2}{PQ}+1\right)\\+\hat{z}\left(e^{-Pz}-e^{-Qz}\right)\left(\frac{(ik\sin\theta)^2}{PQ}+\frac{Q}{P}\right)\frac{ik\sin\theta}{P}\end{array}\right\}e^{2ikx\sin\theta}\end{aligned}$$

$$\begin{aligned}\vec{\Sigma}_3=&\frac{i\omega e}{m}\frac{1}{c^2}\frac{\omega_p^2}{\upsilon_F^2}\left\{\left[1-\frac{\omega_p^2}{\omega\left(\omega+\frac{i}{\tau}\right)}\right]\frac{ik\sin\theta}{QP}\right\}^2\\&\cdot E_0^2\Im^2e^{-2Qz}\left(ik\sin\theta\hat{x}-Q\hat{z}\right)e^{2ikx\sin\theta}\end{aligned}$$

$$\begin{aligned}\vec{\Sigma}_4=&\frac{2i\omega e}{m}\frac{\omega_p^2}{c^2}\frac{1}{\omega\left(\omega+\frac{i}{\tau}\right)}\frac{1}{P}\left\{1-\frac{\omega_p^2}{\omega\left(\omega+\frac{i}{\tau}\right)}\right\}\frac{\omega^2}{c^2}\\&\cdot E_0^2\Im^2e^{-Pz}\left\{\begin{array}{l}\hat{z}\left[\frac{(ik\sin\theta)^2}{PQ}e^{-Qz}+e^{-Pz}\right]\\-\hat{x}\left[e^{-Pz}-e^{-Qz}\right]\frac{ik\sin\theta}{P}\end{array}\right\}e^{2ikx\sin\theta}\end{aligned}.$$

Since the x – dependence of the fields must match that of the currents, the second order equation reduces to two coupled ordinary linear differential equations for the electric field components E_{2x} and E_{2z}:

$$\begin{aligned}&\partial_z^2\mathrm{E}_{2x}+\frac{1}{c^2}\left\{4\omega^2-\frac{2\omega}{2\omega+\frac{i}{\tau}}\left[\omega_p^2+\frac{\upsilon_F^2}{3}(2K)^2\right]\right\}\mathrm{E}_{2x}\\&-\left\{1-\frac{2\omega}{2\omega+\frac{i}{\tau}}\frac{\upsilon_F^2}{3c^2}\right\}(2iK)\partial_z\mathrm{E}_{2z}(z)=\frac{\frac{1}{2}\Sigma_x(z)}{-2i\omega+\frac{1}{\tau}}\end{aligned}\qquad(2a)$$

$$\begin{aligned}&\left[\frac{2i\omega}{c^2\left(-2i\omega+\frac{1}{\tau}\right)}\left[-\frac{\upsilon_F^2}{3}\partial_z^2+\omega_p^2\right]+\frac{4\omega^2}{c^2}-(2K)^2\right]\mathrm{E}_{2z}\\&-\left[1+\frac{2i\omega}{\left(-2i\omega+\frac{1}{\tau}\right)}\frac{\upsilon_F^2}{3c^2}\right](2iK)\partial_z\mathrm{E}_{2x}(z)=\frac{\frac{1}{2}\Sigma_z(z)}{-2i\omega+\frac{1}{\tau}}\end{aligned},\qquad(2b)$$

where $K = k\sin\theta$ and the inhomogeneous terms are known and rather simple functions of z. Since they are quadratic combinations of the linear quantities, which in turn are decaying exponentials into the metal bulk, we can write

$$\begin{pmatrix}\Sigma_x\\ \Sigma_z\end{pmatrix} = \frac{i\omega e}{m}\begin{bmatrix}\begin{pmatrix}H_x\\ H_z\end{pmatrix}e^{2Pz} + \begin{pmatrix}J_x\\ J_z\end{pmatrix}e^{(P+Q)z}\\ +\begin{pmatrix}L_x\\ L_z\end{pmatrix}e^{2Qz}\end{bmatrix} \cdot E_0{}^2\Im^2 e^{2iKx}e^{-2i\omega t} ,$$

where the coefficients in the large brackets are given in the Appendix. This in turn implies that the solution to the field equations will have a particular part and a homogeneous part, where the former must have the form

$$\vec{\mathrm{E}}_{p2} = \begin{pmatrix}\mathrm{E}_{2x}{}^{(H)}\\ \mathrm{E}_{2z}{}^{(H)}\end{pmatrix}e^{2Pz} + \begin{pmatrix}\mathrm{E}_{2x}{}^{(J)}\\ \mathrm{E}_{2z}{}^{(J)}\end{pmatrix}e^{(P+Q)z} + \begin{pmatrix}\mathrm{E}_{2x}{}^{(L)}\\ \mathrm{E}_{2z}{}^{(L)}\end{pmatrix}e^{2Qz}$$

and the latter

$$\vec{\mathrm{E}}_{h2} = \mathbb{A}\begin{pmatrix}-P_2\\ 2iK\end{pmatrix}e^{P_2 z} + \mathbb{B}\begin{pmatrix}2iK\\ Q_2\end{pmatrix}e^{Q_2 z} .$$

Here the decay terms P_2 and Q_2 are given by Eq. (1) with the replacements $\omega \to 2\omega$ and $K \to 2K$. Note that $P_2 \neq 2P$ and $Q_2 \neq 2Q$ in general. Matching the exponentials gives a set of matrix equations for the particular solution fields of the form

$$\begin{aligned}
\mathrm{M}(2P)\begin{pmatrix}\mathrm{E}_{s2x}{}^{(H)}\\ \mathrm{E}_{s2z}{}^{(H)}\end{pmatrix} &= -\Lambda\frac{e}{2m}E_0{}^2\Im^2\begin{pmatrix}H_x\\ H_z\end{pmatrix}\\
\mathrm{M}(P+Q)\begin{pmatrix}\mathrm{E}_{s2x}{}^{(J)}\\ \mathrm{E}_{s2z}{}^{(J)}\end{pmatrix} &= -\Lambda\frac{e}{2m}E_0{}^2\Im^2\begin{pmatrix}J_x\\ J_z\end{pmatrix}\\
\mathrm{M}(2Q)\begin{pmatrix}\mathrm{E}_{s2x}{}^{(L)}\\ \mathrm{E}_{s2z}{}^{(L)}\end{pmatrix} &= -\Lambda\frac{e}{2m}E_0{}^2\Im^2\begin{pmatrix}L_x\\ L_z\end{pmatrix}
\end{aligned} .$$

where

$$\mathrm{M}(\Pi) = \begin{pmatrix}\Pi^2 + \frac{4\omega^2}{c^2} - \Lambda\frac{\omega_p{}^2}{c^2} - \Lambda\beta^2(2K)^2 & -\left\{1-\Lambda\beta^2\right\}(2iK)\Pi\\ -\left\{1-\Lambda\beta^2\right\}(2iK)\Pi & \Lambda\beta^2\Pi^2 + \frac{4\omega^2}{c^2} - \Lambda\frac{\omega_p{}^2}{c^2} - (2K)^2\end{pmatrix}$$

and

$$\Lambda = \frac{2\omega}{2\omega + \frac{i}{\tau}} \quad \beta^2 = \frac{\upsilon_F{}^2}{3c^2} \quad \Xi^2 = \frac{4\omega^2}{c^2} - \Lambda\frac{\omega_p{}^2}{c^2} \quad \eta = \Lambda\beta^2 .$$

The matrix $\mathrm{M}(\Pi)$ arises from the linear (vector) operator on the left side of Eqs. (2a) and (2b). Such operators always appear in this type of perturbation expansion.

For the homogeneous solution we must solve another boundary value problem. Equations (2a) and (2b) describe the electric field inside the metal; outside in the vacuum this field has the form

$$\vec{\mathrm{E}}_{2\mathrm{vac}}(z) = \mathbb{C}E_0{}^2\Im^2\begin{pmatrix}-ip_2\\ 2iK\end{pmatrix}e^{ip_2 z} = 2i\frac{\omega}{c}\mathbb{C}E_0{}^2\Im^2\begin{pmatrix}-\cos\theta\\ \sin\theta\end{pmatrix}e^{ip_2 z} ,$$

where $\mathbb{C}$ is yet another unknown. Clearly it is the most important quantity to determine, since it contains information about the fields leaving the metal and headed for the receive antenna. Note that it is an outgoing wave in this geometry with no incoming wave, which is what we expect for a field radiation by the nonlinear currents. Matching tangential electric and magnetic fields, and again ensuring that the normal current vanishes at the surface, we derive expressions for all the unknowns. The most important one, $\mathbb{C}$, is given by

$$\mathbb{C} = i\frac{c}{2\omega}\frac{1}{\Delta}\left\{\begin{matrix}\left[P_2Q_2 - 4K^2\left(1-\mathrm{E}_2\right)\right]\Psi\\ +\frac{4\omega^2}{c^2}\mathrm{E}_2\left[Q_2\Theta - 2iK\left(1-\mathrm{E}_2\right)\Phi\right]\\ +2iK\mathrm{E}_2\mathrm{X}\end{matrix}\right\} ,$$

where

$$\mathrm{E}_2 = 1 - \frac{\omega_p{}^2}{2\omega\left(2\omega + \frac{i}{\tau}\right)} \qquad p_2 = \frac{\omega}{c}\cos\theta$$

and

$$\begin{pmatrix}\Theta\\ \Phi\end{pmatrix} = -\Lambda\frac{e}{2m}\left\{\begin{matrix} \mathrm{M}^{-1}(2P)\begin{pmatrix}H_x\\ H_z\end{pmatrix} + \mathrm{M}^{-1}(P+Q)\begin{pmatrix}J_x\\ J_z\end{pmatrix} \\ +\mathrm{M}^{-1}(2Q)\begin{pmatrix}L_x\\ L_z\end{pmatrix}\end{matrix}\right\}$$

$$\begin{aligned} \mathrm{X} &= \frac{-\Lambda\frac{e}{2m}}{4P^2 - Q_2{}^2} 2P\left(2iKH_x + 2PH_z\right) \\ &+ \frac{-\Lambda\frac{e}{2m}}{(P+Q)^2 - Q_2{}^2}(P+Q)\left(2iKJ_x + [P+Q]J_z\right) \qquad (4a) \\ &+ \frac{-\Lambda\frac{e}{2m}}{4Q^2 - Q_2{}^2} 2Q\left(2iKL_x + 2QL_z\right) \end{aligned}$$

$$\Psi = \frac{-\Lambda\frac{e}{2m}}{(P+Q)^2 - P_2{}^2}\left([P+Q]J_x - 2iKJ_z\right) \quad . \qquad (4b)$$

$$\Delta = \left(P_2 - ip_2\mathrm{E}_2\right)Q_2 - 4K^2\left(1-\mathrm{E}_2\right)$$

It is worth noting that the denominator Δ has a complex pole at a frequency where $p_2 = -i\wp_2$, i.e., where

$$\left(P_2 + \wp_2\mathrm{E}_2\right)Q_2 - 4K^2\left(1-\mathrm{E}_2\right) = 0 \quad . \qquad (3)$$

This expression was derived by J. Crowell and R. Ritchie in 1968 [9] as the dispersion relation for surface plasmons at a metal surface. The existence of such poles in the linear reflectivity of a dielectric and their association with surface waves is well known; the appearance of this quantity Eq. (3) indicates that whenever the second harmonic can give rise to this type of surface plasmon the value of $\mathbb{C}$ will be large. However, since the frequencies where this can occur are in the optical range, this result is of little interest in the present context

V. Properties of Second Harmonic Emission

The weakness of the second-harmonic emission from the metal surface can be traced to two factors. The first is the presence of the squared transmission coefficient $\Im^2$ in the field *amplitude*, which implies a power emission proportional to $|\Im|^4$. Because $|\Im|$ in copper is only $\sim 10^{-4}$, we have an immediate loss of 160 dB in the amount of incident power converted to second-harmonic power. The second factor is the surprising inefficiency of the nonlinear mechanisms in the metal, as measured by the extreme smallness of the coefficient $\mathbb{C}$, which when evaluated numerically for copper gives rise to an additional decrease of $\sim 10^{-8}$ in the amplitude, so the net emission of power at the target surface is more than 320 dB down from the incident power. In Fig. 2 we plot the magnitude of $\mathbb{C}$ for copper versus angle of incidence of the fundamental. Careful algebra shows that $\mathbb{C} = 0$ for normal emission, increasing to $\sim 10^{-8}$ at nearly grazing emission. Following Ishimaru's definition [10], the factor $\left|\mathbb{C}\Im^2\right|^2 E_0{}^2$ is the (power-dependent) RCS per unit area for the second-harmonic signal.

A third factor that inhibits the use of nonlinear radar in general arises from modification of the radar equation for nonlinear radar, which we discussed in detail in Ref. [11]. According to the usual arguments [12], a field radiated by an transmitting antenna and incident on a target a distance *R* from the latter suffers a loss proportional to R^{-1} (which in our case here is contained in the factor E_0). When the target is nonlinear, this field must be squared in the second harmonic current, producing an R^{-2} dependence in the radiated field $\vec{E}_{2\mathrm{vac}}$. Counting the return-trip loss of another factor of R^{-1}, it is clear that the field received by the antenna (monostatic case) is down by R^{-3}, so the power is down by R^{-6} [11].

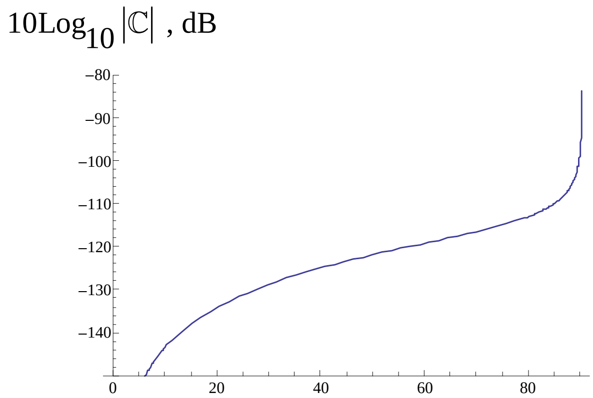

FIG. 2. **Second-harmonic RCS**

It is easily shown that the second harmonic power is radiated collinearly with the linearly reflected signal. This follows from the replacements $\omega \to 2\omega$ and $K \to 2K$ in the various expressions given above, which implies that the angle of emission is identical with the incident angle. This is ultimately due to the evanescent nature of the fields inside the metal.

VI. Conclusion

We have shown that the strong suppression of second-harmonic emission from a metal surface is a consequence of two things: the inability of the external fields to penetrate the metal surface due to linear reflectivity, and the inefficiency of

nonlinear processes in the metal associated the with free-carrier (i.e., plasma) mechanisms. These results exacerbate difficulties we have discussed in Ref. [11], and suggest that the uses of nonlinear radar may be few. However, it is worth noting that because metals are highly detectable to linear radar, there are ongoing efforts to render objects invisible by making them nonmetallic, e.g., plastic guns. Such objects may be much more visible to probes based on the nonlinear techniques.

VII. APPENDIX

The coefficients of the inhomogeneous terms that appear in Eqs. (4a) and (4b) are:

$$\begin{pmatrix} H_x \\ H_z \end{pmatrix} = -\lambda \mathrm{E} \frac{1}{c^2 P} \frac{\omega_p{}^2}{c^2} \begin{pmatrix} \frac{iK}{P} \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} J_x \\ J_z \end{pmatrix} = \frac{\omega_p{}^2}{c^2} \begin{pmatrix} \frac{\lambda}{\omega^2} \left\{ \lambda - 1 + (\lambda+1)\left(\frac{K^2}{P^2} + \frac{Q}{P}\right) - (3\lambda+1)\frac{K^2}{PQ} \right\} iK \\ -\frac{1}{c^2 P^2} \mathrm{E} \left\{ \frac{1}{\beta^2}(1+2\lambda) - \lambda \right\} \frac{K^2}{Q} \end{pmatrix}$$

$$\begin{pmatrix} L_x \\ L_z \end{pmatrix} = -\frac{1}{c^2 P^2} \mathrm{E} \frac{1}{\beta^2} \frac{K^2}{Q} \frac{\omega_p{}^2}{c^2} \left\{ \frac{1}{3}\mathrm{E} - 1 - 2\lambda \right\} \begin{pmatrix} \frac{iK}{Q} \\ 1 \end{pmatrix} \quad ,$$

where

$$\lambda = \frac{\omega}{\omega + \frac{i}{\tau}} \; , \quad \mathrm{E} = 1 - \frac{\omega_p{}^2}{\omega\left(\omega + \frac{i}{\tau}\right)} \quad .$$

The remaining quantities are defined in the main text.

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